Analysis and Measurement of Intrinsic Noise in Op Amp Circuits
Part XI: Photodiode Amplifier Noise
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This TechNote focuses on noise analysis and simulation of photodiode amplifier circuits. Additionally, the process for optimizing the feedback compensation capacitor for output noise and stability will be a key focus.

Short Introduction to Photodiodes

To understand how to properly configure and analyze a photodiode in an amplifier configuration, it is important to understand some basic fundamentals on photodiode operation. A photodiode is a semiconductor device that is used to convert light to electrical current or voltage. Fig. 11.1 shows a simple pn photodiode consisting of n and p doped semiconductor material. With no bias applied to the diode, the free electrons from the n region combine with the free holes in the p region to create a depletion region. The depletion region is charged positively in the n material and negatively in the p material, so it develops an e-field.

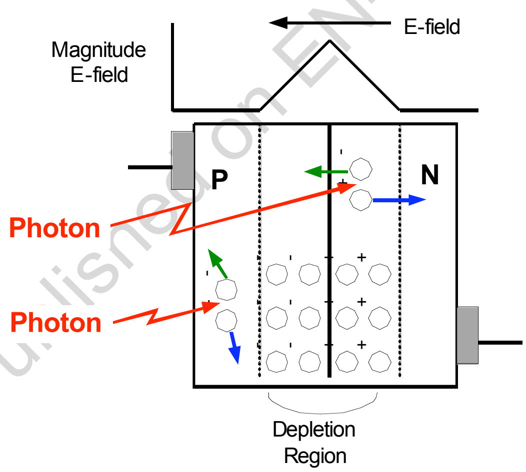
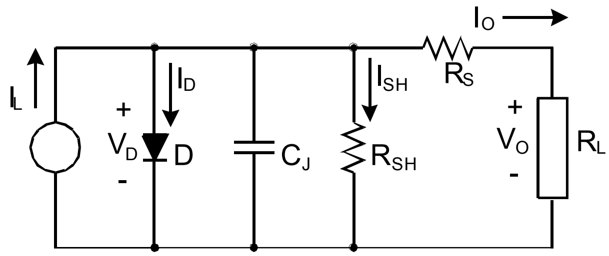


Fig. 11.1: Simplified Semiconductor Model of Photodiode

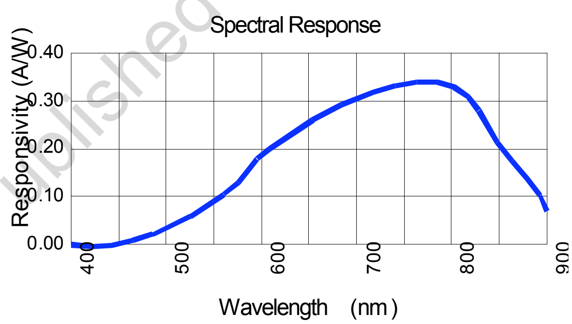
The schematic shown overleaf in Fig. 11.2 is the model for a photodiode. The different components in the model are normally given in the photodiode's data sheet. The junction capacitance (Cj), shunt resistance (Rsh), and dark current (Id) are key parameters used in noise analysis.



I_L : Current generated by the incident light
I_D : Diode Current
C_j : Junction capacitance
R_{sh} : Shunt resistance
R_s : Series resistance
I_{SH}: Shunt resistance current
V_D : Diode Voltage
I_o : Output current
V_o : Output voltage

Fig. 11.2: Photodiode Electrical Model

The purpose of a photodiode is to convert light to current. Fig. 11.3 shows that the responsivity of a photodiode to light is affected by the wavelength. Different types of photodiodes are designed and optimized to respond at specific wavelengths.



I_L : Current generated by the incident light

$I_L = r_\phi \phi_e$

r_ϕ is the diode's flux responsivity

ϕ_e is the radiant flux energy in Watts

Fig. 11.3: Photodiode Responsively Vs Wavelength

The junction capacitance is a key parameter used in noise analysis. Typically smaller junction capacitance leads to lower output noise. Increasing the reverse bias voltage on the diode decreases the junction capacitance. Therefore, in some applications increasing the reverse bias voltage is necessary to reduce the overall output noise. The equation shown in Fig. 11.4 illustrates the mathematical relationship between junction capacitance and reverse voltage.

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{\phi_B}}}$$

C_{j0} is the photodiode capacitance at zero bias

ϕ_B is the built-in voltage of the diode junction

V_R is the reverse bias voltage

Fig. 11.4: Photodiode Junction Capacitance Vs Reverse Bias Voltage

Fig. 11.5 shows how the diode characteristic curves are shifted by applied light. With no light applied, the photodiode acts as a conventional rectifier. Increasing applied light shifts the curve downward on the current axis.

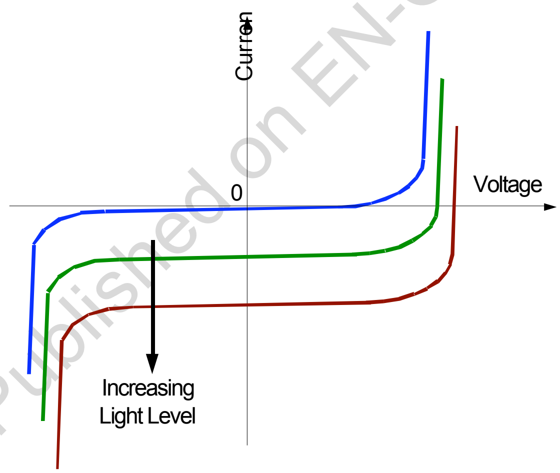


Fig. 11.5: Photodiode V–I Characteristics

Simple Transimpedance Amplifier

The circuit shown overleaf in Fig. 11.6 is referred to as a transimpedance amplifier and is the most commonly-used photodiode amplifier configuration. The analysis contained within this TechNote is based on this simple transimpedance topology.

The output voltage is calculated by multiplying the input current from the photodiode by the feedback resistor Rf.

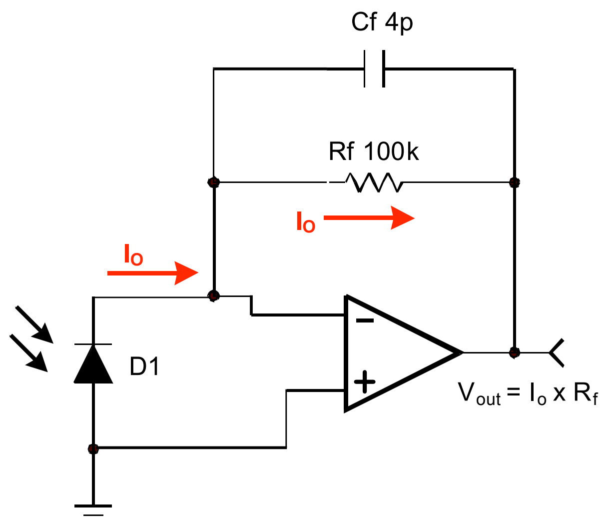


Fig. 11.6: Common Transimpedance Amplifier

Bandwidth for Simple Transimpedance Amplifier

The transimpedance amplifier noise bandwidth and noise gain shown in Fig. 11.6 are the two important factors that contribute to the overall output voltage noise. However, since the overall output signal resolution of a transimpedance amplifier is determined by the ratio of the output signal to the output noise or commonly known as the signal-to-noise ratio (SNR), it is important to understand the bandwidth limit for the photodiode signal (signal bandwidth). The signal bandwidth (f_p) is shown in Fig. 11.7. Note that the signal bandwidth is limited by R_f and C_f.

$$C_f := 4\text{pF}$$

$$R_f := 100\text{k}\Omega$$

$$f_p := \frac{1}{2\pi \cdot R_f \cdot C_f} = 398\times 10^3 \text{ Hz}$$

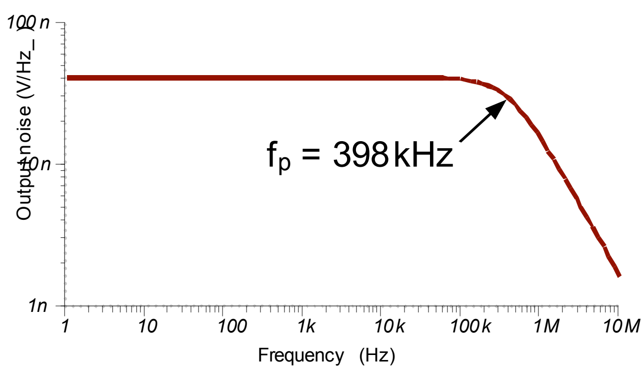


Fig. 11.7: Bandwidth for Simple Transimpedance Amplifier

Noise Model for Simple Transimpedance Amplifier

Analysis of the photodiode amplifier's output noise can be broken into three subsections: the photodiode, op amp, and resistor. Each of these three subsections can be modeled as a separate noise source that will be combined to compute the total output noise. The input (Copa) and feedback (Cf) capacitance and the shunt (Rs) and feedback (Rf) resistance in this circuit significantly affect the noise because they shape the noise gain curve.

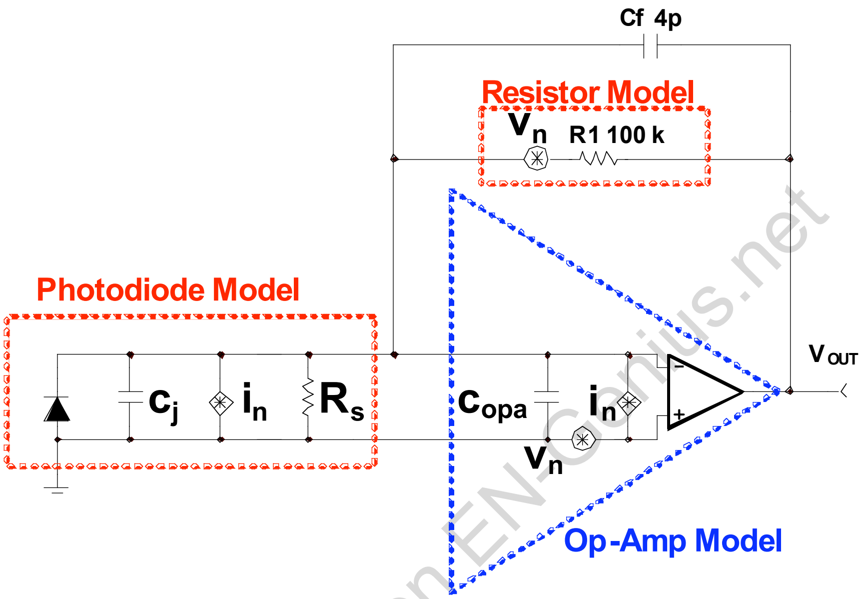


Fig. 11.8: Noise Model for Simple Transimpedance Amplifier

Photodiode Current Noise

The current noise of a photodiode is the root sum square of three different noise sources: the thermal (Johnson) noise of the shunt resistance, the dark current shot noise, and the shot noise of the light current. Normally we consider the thermal noise of a resistor to be a voltage noise. However, for the analysis of a photodiode, it is convenient to analyze the thermal noise as a current.

$$i_j = \sqrt{\frac{4k_b \cdot T_n}{R_{sh}}} \quad \text{Thermal (Johnson Noise)}$$

k_b Boltzmann constant 1.38*10 $^{-23}$ J/K

T_n Temperature in Kelvin (25C)

R_{sh} Shunt Resistance in photodiode

Fig. 11.9: Photodiode Thermal Current Noise Spectral Density

Photodiode noise also contains shot noise. Shot noise is proportionate to, and is only present, when dc current flows. Two types of shot noise are present in the photodiode circuit. One is caused by the current that flows when light is applied to the photodiode (IL). The other noise source is caused by the dark current (ID). Fig. 11.10 shows the photodiode shot noise equations.

$$i_{sD} = \sqrt{2q \cdot I_D} \quad \text{Shot noise (dark)}$$

$$i_{sL} = \sqrt{2q \cdot I_L} \quad \text{Shot noise (w. Light)}$$

q Electron Charge $1.6*10^{-19}$ C

$\mathbf{I_D}$ Dark Current in photodiode

$\mathbf{I_L}$ Photo current in photodiode

Fig. 11.10: Photodiode Shot Current Noise Spectral Density

The three current sources from the diode can be added using the root sum of the square (see Fig. 11.11) to produce the overall noise of the photodiode, i_{n_diode}. The total noise current from the photodiode flows through the feedback resistor Rf and forms an overall rms voltage noise at the output of the transimpedance amplifier (see Fig. 11.12). It is important to note that the bandwidth limit for the current noise is the signal bandwidth of the transimpedance amplifier multiplied by the brick wall correction factor (noise bandwidth).

$$i_{n\_diode} = \sqrt{i_j^2 + i_{sD}^2 + i_{sL}^2} \quad \text{Total Diode Current Noise}$$

$\mathbf{i_j}$ Thermal (Johnson Noise)

$\mathbf{i_{sD}}$ Dark Current in photodiode

$\mathbf{i_{sL}}$ Photo current in photodiode

Fig. 11.11: Total Photodiode Current Noise Spectral Density

$$i_{n\_total} = \sqrt{i_{n\_opa}^2 + i_{n\_diode}^2} \quad \text{Total Noise Current}$$

$$BW_n = K_n \cdot f_p \quad \text{Noise bandwidth (brick wall filter)}$$

$$E_{noI} = i_{n\_total} \cdot R_f \cdot \sqrt{BW_n} \quad \text{Voltage noise at output from total current noise}$$

Fig. 11.12: Rms Noise Voltage from Photodiode Current Noise

Thermal Noise from Rf

The thermal noise of the feedback resistor can be calculated using the equations shown in Fig. 11.13. Note that the bandwidth limit for the thermal noise is the signal bandwidth of the transimpedance amplifier multiplied by the brick wall correction factor (noise bandwidth).

$E_{noR} = \sqrt{4k_b \cdot T_n \cdot R_f \cdot BW_n}$	Thermal noise at output
$BW_n = K_n \cdot f_p$	Noise bandwidth (brick wall filter)
$k_b = 1.38 \cdot 10^{-23} \frac{J}{K}$	Boltzmann constant
T_n	Temperature in Kelvin
f_p	Transconductance bandwidth
K_n	Brick Wall Factor

Fig. 11.13: Thermal Noise from Feedback Resistor in Transimpedance Amplifier

Noise from Op Amp Voltage Noise Source

The final contribution to the output noise to consider is the intrinsic voltage noise of the op amp itself, which is complicated by the ac noise gain, ie, the gain seen by the noise voltage signal source. In this example, Cf and Cin cause a peak in the noise gain curve that significantly affects the total output noise. The circuit in Fig. 11.14 shows the key components that affect the noise gain.

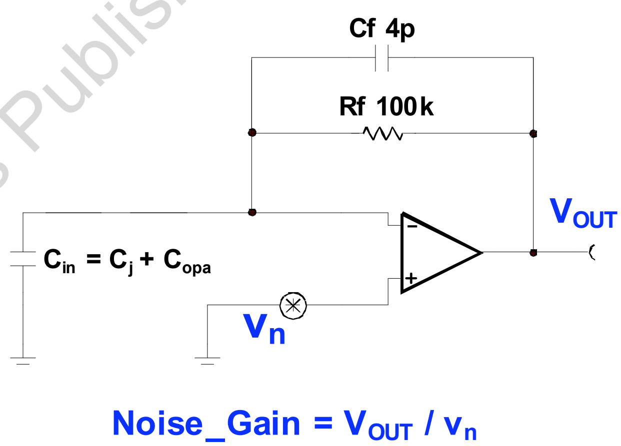
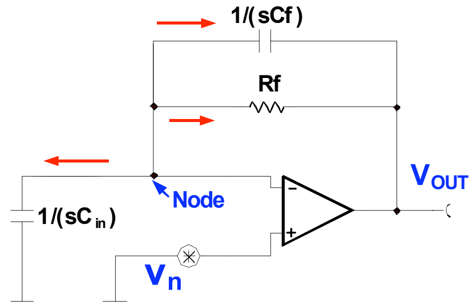


Fig. 11.14: Noise from Op Amp Voltage Noise Source

To compute the noise gain for the photodiode amplifiers, we perform a nodal analysis at the summing junction of the inverting amplifier input yielding three current paths (red markers on Fig. 11.15). To complete this analysis we need to consider the impedance of the capacitors: we represent the impedance of a capacitor as 1/(sCf), where s is jω.



$$\text{Noise\_Gain} = V_{OUT} / v_n$$

Fig. 11.15: Nodal Analysis for Noise Gain Vs Frequency

Fig. 11.16 shows the algebra for the nodal analysis. The first equation in this figure is taken directly from the node. Note that the equation has three terms for each of the three current paths. The first equation is rearranged into the noise gain transfer function. The numerator of this transfer function contains a zero and the denominator contains a pole. The zero (f_z) and pole (f_p) have significant effects on the transfer function. The equations for the pole and zero are shown at the bottom of Fig. 11.16.

Nodal Analysis on transimpedance amp

$$\frac{V_n}{\frac{1}{s \cdot C_{in}}} + \frac{(V_n - V_{out})}{R_f} + \frac{V_n - V_{out}}{\frac{1}{s \cdot C_f}} = 0$$

Solve for noise gain Vout / Vn

$$\frac{V_{out}}{V_n} = \frac{R_f \cdot (C_f + C_{in}) \cdot s + 1}{C_f \cdot R_f \cdot s + 1}$$

The numerator contains a **Zero**

$$f_z = \frac{1}{2\pi\, R_f \cdot (C_f + C_{in})}$$

The denominator contains a **Pole**

$$f_p = \frac{1}{2\pi\, R_f \cdot C_f}$$

Fig. 11.16: Solve for Pole and Zero Using Nodal Analysis

Noise gain versus frequency is displayed in Fig. 11.17. Note that noise gain is 0 dB until the zero (f_z). Between the zero (f_z) and the pole (f_p) the noise gain rises at 20 dB/decade. The pole and zero cancel so that the noise gain flattens out. The flat region continues until it intercepts the Aol curve at f_i, then rolls off with Aol. The magnitude of the flat region between f_p and f_i is dependent on C_{IN} and C_f. The flat region between f_p and f_i is called the noise gain peak. Each frequency transition point is useful in deriving equations for the total noise.

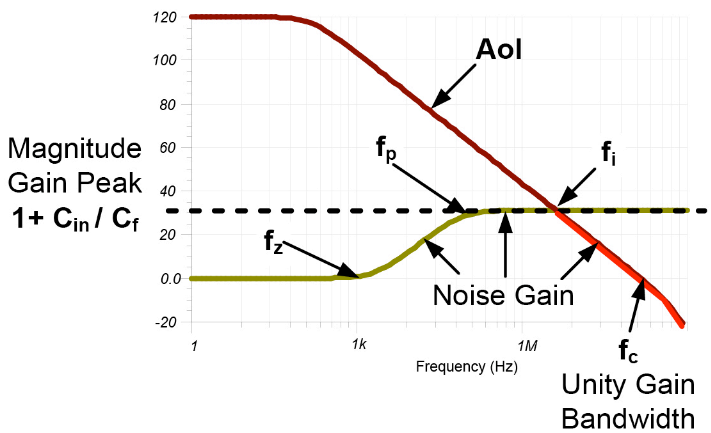


Fig. 11.17: Noise Gain Vs Frequency

Fig. 11.18 gives some of the key equations from the noise gain curve in Fig. 11.17. The equation for f_i shows where the noise gain curve intercepts the Aol curve. The equation for GPM gives the magnitude of the noise gain peak.

$$f_i = \frac{C_f}{C_i + C_f} \cdot f_c$$

Intersection of the noise gain curve with the Aol Curve

$$f_c$$

Unity Gain Bandwidth from Op-Amp Data Sheet

$$GPM = 1 + \frac{C_{in}}{C_f}$$

Gain Peak Magnitude

Fig. 11.18: Key Equations for Noise Gain Curve

Fig. 11.19 shows the op amp voltage noise curve in red, the noise gain in blue, and the output noise in green. The key here is to understand that the op amp noise curve is multiplied by the noise gain curve to produce the output noise curve. To find the total rms output noise we must integrate the output noise curve.

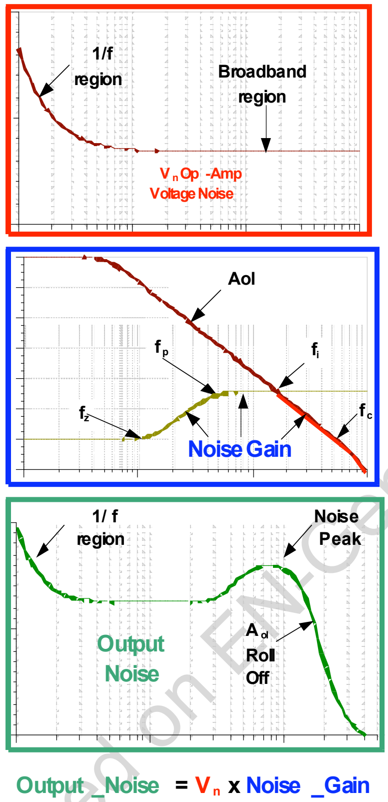


Fig. 11.19: Output Noise Equals Input Noise x Noise Gain

Fig. 11.20 shows a circuit that can be used to generate the noise gain, Aol, and current to voltage gain. A SPICE ac sweep is used to generate curves at the test points VF1, VF2, and VF3. These signals are post-processed using the formulas from Fig. 11.21. Note that the 1TH inductor is used to break the feedback loop from an ac perspective, but allows for a dc connection. The 1TF capacitor allows the signal source VG1 for ac coupling into the loop at extremely low frequencies. These values are not practical for any physical circuit, but work well for this SPICE curve generation technique.

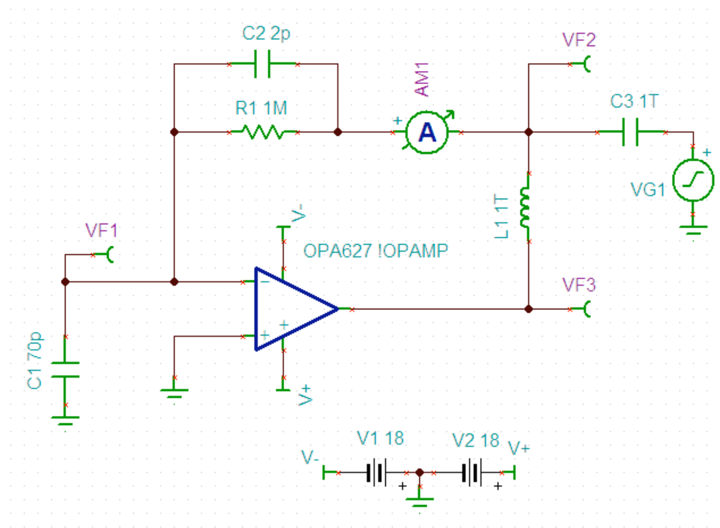


Fig. 11.20: SPICE Circuit to Find Aol and Noise Gain

Fig. 11.21 shows how to start an "ac transfer characteristic" using TINA SPICE. After starting the ac transfer characteristic you need to enter the start and end frequency as required for your application. The ac transfer characteristic will create a curve for each of the test points and meters in the circuit.

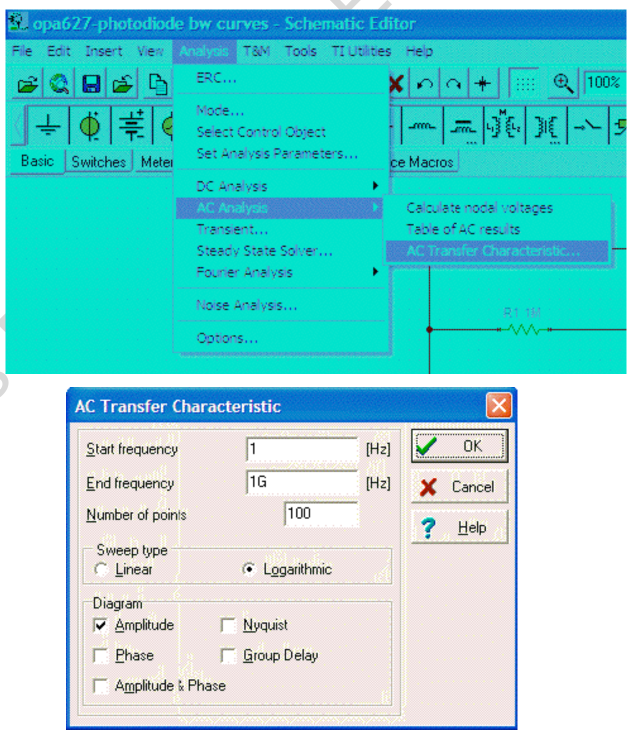


Fig. 11.21: Running Ac Transfer Character in TINA SPICE

The post-processing feature of SPICE enables the creation of the key transimpedance curves (Fig. 11.22) using math on the curves generated by the ac transfer characteristic.

$$A_{ol} = \frac{VF3}{VF1}$$

$$Noise\_Gain = \frac{1}{\beta} = \frac{VF2}{VF1}$$

$$I\_to\_V\_Gain = \frac{VF3}{AM1}$$

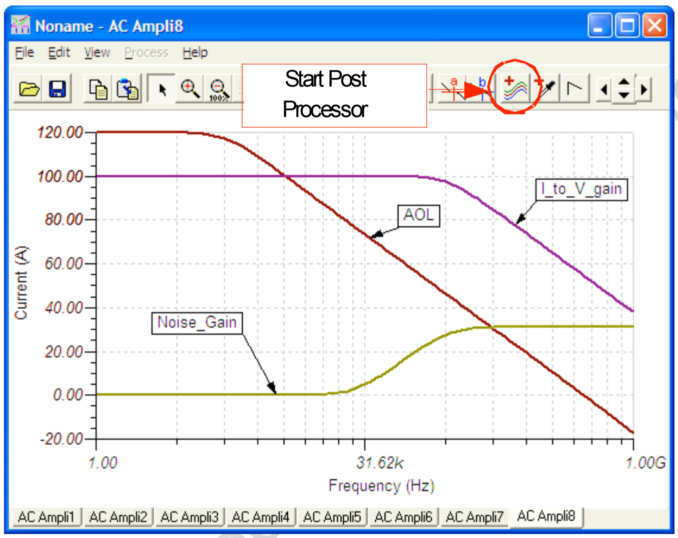


Fig. 11.22: Noise Gain and Aol in Ac Plot

Fig. 11.23 shows the Aol, noise gain, and current-to-voltage curves generated by entering the equation into the post processor.

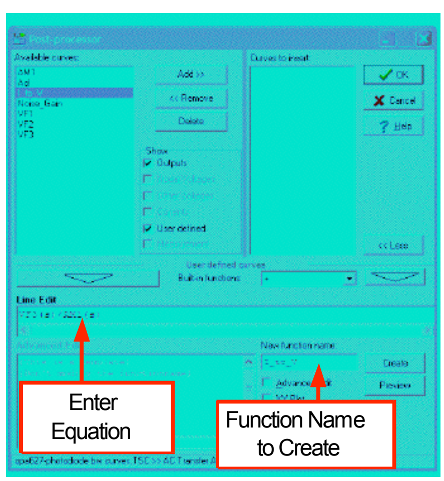


Fig. 11.23: Creating Noise Gain and Aol with Post Processor

The output noise spectral density is the product of the noise gain and the voltage noise spectral density (see Fig. 11.24). Note that region 2 is a constant between regions 1 and 3 where no term dominates.

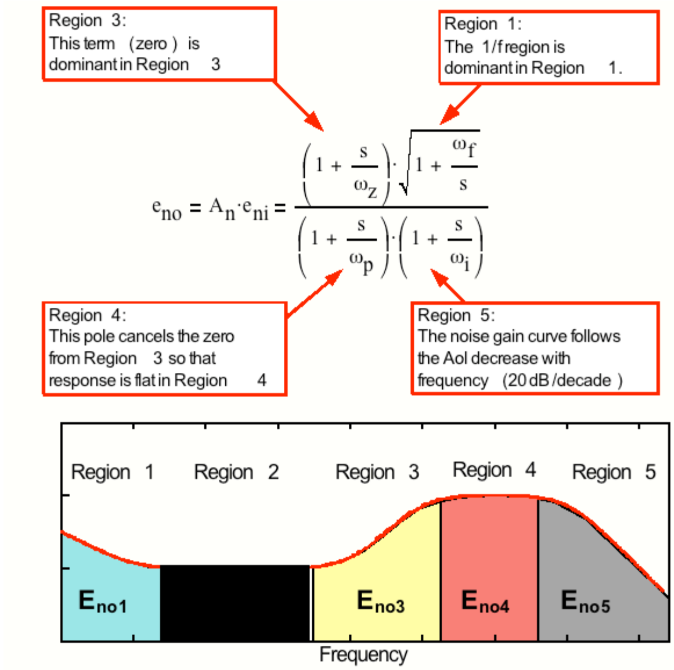


Fig. 11.24: Noise Gain Transfer Function

$$E_{noe1} = \sqrt{\int_{f_L}^{f_f} \frac{e_{nif}^2 f_f}{f}\,df} = e_{nif}\cdot\sqrt{f_f \cdot \ln\left(\frac{f_f}{f_L}\right)}$$

$$E_{noe2} = \sqrt{\int_{f_f}^{f_z} e_{nif}^2\,df} = e_{nif}\cdot\sqrt{(f_z - f_f)}$$

$$E_{noe3} = \sqrt{\int_{f_z}^{f_p} \frac{e_{nif}^2 f^2}{f_z^2}\,df} = \frac{e_{nif}}{f_z}\cdot\sqrt{\frac{f_p^3 - f_z^3}{3}}$$

$$E_{no4} = \sqrt{\int_{f_p}^{f_i} e_{nif}^2\cdot\left(\frac{C_{in} + C_f}{C_f}\right)^2 df} = e_{nif}\left(\frac{C_{in} + C_f}{C_f}\right)\cdot\sqrt{(f_i - f_p)}$$

$$E_{no5} = \sqrt{\int_{f_i}^{\infty} \frac{e_{nif}^2\cdot f_c^2}{f^2}\,df} = \frac{e_{nif}\cdot f_c}{\sqrt{f_i}}$$

$$E_{noe} = \sqrt{E_{no1}^2 + E_{no2}^2 + E_{no3}^2 + E_{no4}^2 + E_{no5}^2}$$

Fig. 11.25: Total Rms Noise from Noise Voltage in Transimpedance Amplifier

We integrate the power spectral density in each of the five regions and combine the results (rms of the noise components) to get the total noise (see Fig. 11.25).

Fig. 11.26 shows the output noise spectral density with logarithmic and linear x-axes. The linear scale is intended to emphasize that regions 3, 4, and 5 dominate the total noise. Looking at the log scale it is easy to be misled into believing that the 1/f region (R1) could be the dominant source of noise.

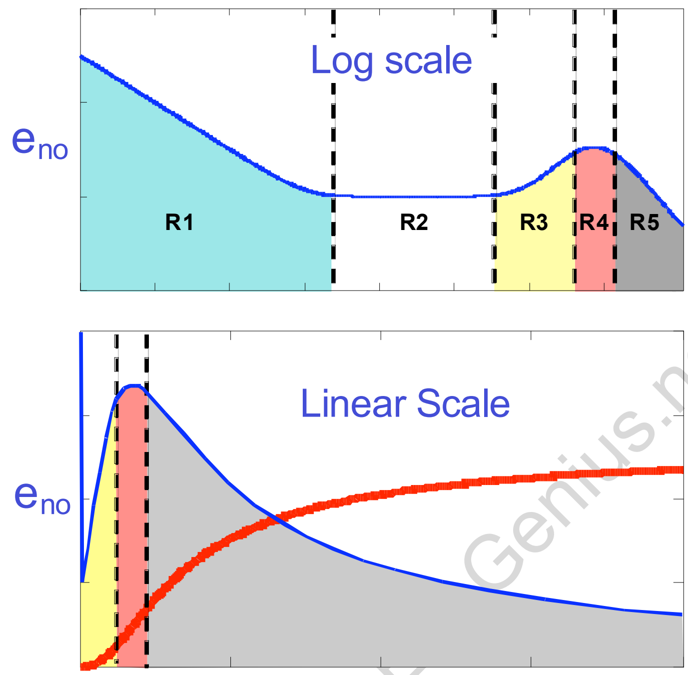


Fig. 11.26: Different Regions in Voltage Noise Curve on Logarithmic/Linear Scales

Total Noise (Op Amp, Diode, and Resistance)

At this point we have determined relationships for all three noise sources in the transimpedance circuit: resistor noise, current noise, and voltage noise. To compute the total output noise we combine these three results using the root sum of the square.

$$E_{no} = \sqrt{E_{noR}^{2} + E_{noI}^{2} + E_{noe}^{2}} \qquad \text{Total Output Noise for the Transimpedance Amp}$$

Fig. 11.27: Total Rms Output Noise for Transimpedance Amplifier

Stability of Transimpedance Amplifier

In addition to shaping the output noise, the impedances connected at the summing node and the photodiode itself determine the transimpedance amplifier's stability. In fact, a photodiode amplifier without feedback capacitance is an inherent differentiator and will not be stable (see Fig. 11.28, overleaf) because the rate of closure (ROC) between the feedback network (1/beta) and the Aol curve is equivalent to 40 dB/decade. From stability/feedback theory, any system that has an ROC >20 dB/decade will not be stable. This description is shown more precisely in Fig. 11.29.

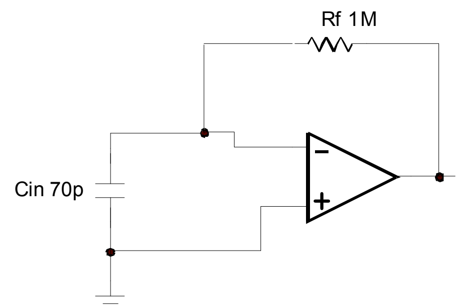


Fig. 11.28: Transimpedance Amplifier Without Cf is Not Stable

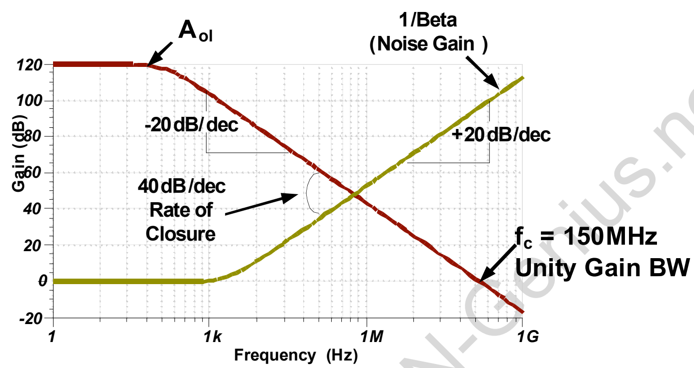


Fig. 11.29: Not Stable with 40 dB Rate of Closure

Another approach to testing stability is to apply a small signal step in photodiode current to the input of the photodiode amplifier circuit. In this case a step input current produces a substantial amount of ringing which in practical circumstances might induce sustained oscillations.

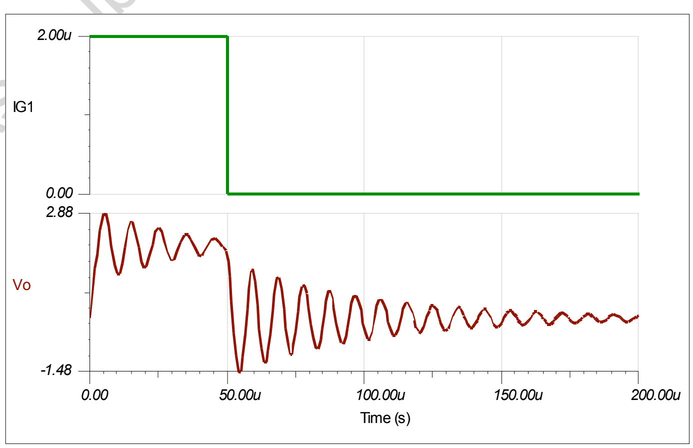


Fig. 11.30: Step Input Shows Stability Issue

The key to stabilizing the photodiode amplifier is to reduce the rate of closure between the noise gain curve and the Aol curve to 20 dB/decade by introducing a pole that flattens out the increasing noise gain before it intersects the Aol curve (see Fig. 11.31). One potential pitfall of this example is that a small change in capacitance, or Aol, can cause the amplifier to become unstable. As a rule of thumb it is advisable to set fp to a frequency at least half to one decade lower for good design margin.

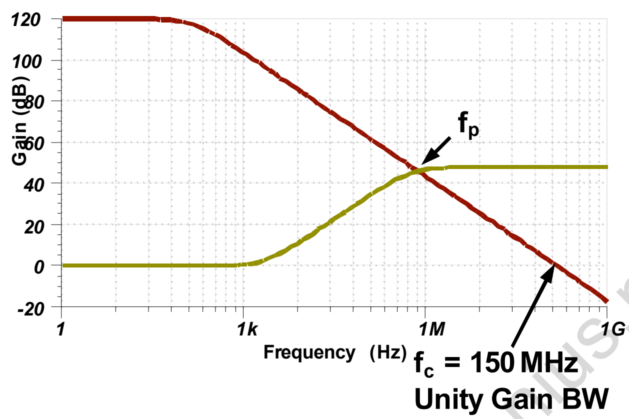


Fig. 11.31: Pole from Cf Stabilizes Circuit

Fig. 11.32 gives the equations for selecting a feedback capacitance that ensure stability. There are two formulas for Cf. The simplified formula assumes that C_{IN} is much greater then Cf. The second equation gives a more exact value that is not dependent on this assumption. Note that these formulas compute the minimum capacitance required for stability. Increasing Cf beyond the minimum ensures design margin.

$f_c = 150\text{MHz}$ Op-amp Unity Gain Bandwidth

$C_{in} = 70\text{pF}$ Total input capacitance

$R_f = 1\text{M}\Omega$ Feedback resistance

$$C_f = \sqrt{\frac{C_{in}}{2\pi \cdot R_f \cdot f_c}} = 272.5\text{fF}$$

Simplified equation for minimum feedback cap Assumes Cin >> Cf

$$C_c = \frac{1}{2\pi \cdot R_f \cdot f_c}$$

Intermediate calculation used in more exact formula

$$C_{fe} = \frac{C_c}{2} \cdot \left(1 + \sqrt{1 + \frac{4C_{in}}{C_c}}\right) = 273.1\text{fF}$$

More exact formula for feedback capacitance

Fig. 11.32: Equations for Selecting Minimum Cf for Stability

Summary

In this TechNote, we have introduced the key equations required for analyzing a photodiode transimpedance amplifier. Special emphasis has been placed on noise peaking caused by interaction from the voltage noise source with the input capacitance. Photodiode basics have also been discussed with emphasis on factors effecting noise.

References

- Jerald Graeme, *Photodiode Amplifiers: OP AMP Solutions*, McGraw-Hill Professional
- Tim Green, *Operational Amplifier Stability*, http://www.en-genius.net
- http://www.hamamatsu.com, Photodiode Technical Information

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